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Electrostatic charge bounds for ball lightning models

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Abstract.
Several current theories concerning the nature of ball lightning predict a substantial electrostatic charge in order to account for its observed motion and shape (Turner 1998 *Phys. Rep.* **293** 1; Abrahamson and Dinniss 2000 *Nature* **403** 519). Using charged soap bubbles as a physical model for ball lightning, we show that the magnitude of charge predicted by some of these theories is too high to allow for the types of motion commonly observed in natural ball lightning, which includes horizontal motion above the ground and movement near grounded conductors. Experiments show that at charge levels of only 10–15 nC, 3-cm-diameter soap bubbles tend to be attracted by induced charges to the nearest grounded conductor and rupture. We conclude with a scaling rule that can be used to extrapolate these results to larger objects and surroundings.

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1. Introduction

Ball lightning is a rare phenomenon of atmospheric physics for which no satisfactory theory has yet been found. One of its most problematic observed characteristics is the way it moves. In a 1966 survey by Rayle [1] covering more than 100 accounts of ball lightning, a majority of the interviewees reported mainly horizontal motion, while only 19% reported vertical motion. Numerous accounts [2, 3] describe ball lightning as often moving more or less parallel to the earth's surface a meter or two above the ground. Any theory claiming to give a satisfactory account of ball lightning must also account for the unusual nature of its observed motion.

Theories of ball lightning fall into two broad categories—external-energy theories and internal-energy theories—with many subdivisions within each [4]. External-energy theories assume that the visible light emission and other energetic manifestations of ball lightning are powered by an external energy source such as radio-frequency or microwave radiation. This idea was first proposed by Kapitsa in 1955 [5]. The object's location and movement in these theories is typically determined by the nature of the assumed electromagnetic field, not solely by the physics or dynamics of the ball itself. On the other hand, internal-energy theories assume that ball lightning is a more or less self-contained structure containing an electrical, chemical, or other type of energy source within it. In these theories, the object's motion must result from interactions between the object and its surrounding environment, where only 'normal' physics is presumed to occur. In this paper, we analyze certain assumptions about electrostatic charge made in two internal-energy theories, and find discrepancies between the consequences of these assumptions and ball lightning's observed behavior.

Several internal-energy theorists have hypothesized a substantial net electrostatic charge for ball lightning in order to explain its motion and shape. Turner [6, 7] has proposed a model of ball lightning which involves an electrochemical reaction among nitrogen compounds in humid air. The model predicts a net positive charge for the overall object of as much as 3 μC. Abrahamson and Dinniss [8] assumed a net electrostatic charge for ball lightning when they hypothesized an interconnected network of slowly oxidizing silicon nanoparticles. According to their theory, repulsion of like charges may be responsible for maintaining the approximately spherical shape of the network.

Since these leading theories of ball lightning include the assumption of a significant electrostatic charge, an experimental investigation of how ball-lightning-like objects move when charged can test the plausibility of this assumption. As we will show below, surprisingly small amounts of net charge are likely to lead to the object's destruction in a short time. If ball lightning does carry a net electrostatic charge, its level must be well below those proposed by Turner, and may be so small as to exert an insignificant influence on its motion compared to other forces.

2. Experimental approach

In this study, we used soap bubbles as a physical model for ball lightning in order to investigate how ball lightning's electrostatic charge can affect its interaction with its environment. Both individual witnesses of the natural phenomenon[1] and numerous studies examining multiple reports [10, 11][2] have noted that ball lightning changes shape, moves, and disappears in ways that sometimes resemble a soap bubble. The structure of ball lightning is in all likelihood much more complex than that of a bubble. However, soap bubbles share with ball lightning the following characteristics: (i) approximately spherical shape, (ii) diameter ranging from 1 cm or less than 50 cm or more, (iii) an apparent density approximating that of ambient air, (iv) a duration in air from a few seconds up to a minute or more (consistent with the observed lifetime of ball lightning [4]), and (v) a surface tension which tends to restore the object's spherical shape when perturbed away from it. In soap bubbles, the surface tension is provided by a solution of soap in water. Such films are stable against perturbations that increase the film area, since an increase in area decreases the soap concentration, leading to higher surface tension which tends to restore the surface to its original area [12]. This is known as the Marangoni effect. In ball lightning, the origin of apparent surface tension is uncertain, although plasmas can show surface-tension-like effects [13]. These characteristics shared by bubbles and ball lightning make bubbles useful as physical models for natural ball lightning, provided any limitations of the model are taken into account.

One such limitation is the fact that an electrostatic charge on a bubble is predominantly surface charge on the conductive liquid film, while charge in ball lightning may be distributed throughout the entire volume of the ball in an unknown way. We will consider two simplified models of the electrostatic conductivity of ball lightning to show how this limitation is probably not a serious one.

In the first model, suppose that ball lightning is an insulator which somehow holds excess charge in a fixed volume distribution. (A physical model for this situation would be a solid sphere of a good insulator such as glass, with excess charge embedded inside it.) Assume that this distribution is given by the function $\rho_V(r, \theta, \psi)$ in $\mathrm{C\,m^{-3}}$ inside a sphere of radius r_o, and $\rho_V = 0$ outside the sphere, where (r, θ, ψ) are the radial distance, polar angle, and azimuthal angle, respectively, of a spherical coordinate system whose origin coincides with the center of the sphere. The electrostatic potential $\phi(r, \theta, \psi)$ everywhere outside the sphere can be expressed as the sum of spherical surface harmonic functions $Y_n(\theta, \psi)$, each weighted by a moment constant p_n,

$$\phi(r, \theta, \psi) = \frac{1}{4\pi\varepsilon} \sum_{n=0}^{\infty} p_n \frac{Y_n(\theta, \psi)}{r^{n+1}}, \tag{1}$$

where ε is the dielectric constant of the medium [14]. The first term in this sum decreases with radius as r^{-1}. It is identical to the field produced by a point charge at the origin whose magnitude equals the net charge of the sphere. The second term decreases as r^{-2} and corresponds to the field of a dipole, the third term to a quadrupole, and so on. Since the higher-order terms decrease more rapidly with increasing r due to the larger exponent of the higher-order terms, at a distance of a few times the sphere radius r_o most of the significant force effects will be a function only of the total charge inside the sphere (the leading term), and not to its particular distribution inside (the higher-order terms). Therefore, if the electrical behavior of a spherical shell of net charge (i.e. a soap bubble) is equated to that of an insulating ball lightning object of equal radius and net charge, the equivalence will be good except for distances less than a few sphere radii, and approximately correct for distances less than this.

Consider a second (and more likely) model for ball lightning's conductivity: that it is a conductor to the extent that an excess electrostatic charge placed on the sphere will be free to move to a steady-state distribution with a time constant on the order of seconds or less. If the charge within a solid conducting sphere obeys Coulomb's inverse-square force law, a well-known result from electrostatics [15] shows that all excess charge will move to the surface of the sphere, and none will remain inside. If this conductivity model is correct, the analogy between charge on ball lightning and the same quantity of charge on a soap bubble will be essentially exact. Even if the Coulomb potential is replaced by the Yukawa potential (also known as the screened Coulomb potential or the Debye–Hückel potential), we reach a similar conclusion. This potential is sometimes used in describing the forces between charged particles in dusty plasmas. The Yukawa potential between two charges of magnitude q is given by

$$\frac{q^2}{4\pi\varepsilon r} \exp\left(-\frac{r}{\lambda_\mathrm{D}}\right), \tag{2}$$

where λ_D is the Debye length in the plasma, a measure of how well the plasma screens isolated charges [16]. Spencer [17] has shown that assuming a Yukawa potential for the charges in a conducting sphere leads to a situation where some of the charge is on the surface and the remainder is distributed uniformly inside. While this distribution differs from the soap bubble's surface-only distribution, the argument above given for the insulating sphere applies equally well to a sphere with uniform charge inside, in that significant differences between the bubble case and the ball-lightning case will appear only for distances within a few sphere radii. Therefore, whatever the details of conductivity for electrostatic charges within ball lightning, we believe our experimental results are likely to be a good model of the forces on an actual ball-lightning object for which electrostatic forces are an important, if not wholly determinative, factor in its motion.

Our experimental setup is shown in figure 1. A Van de Graaff generator 48 cm high with a spherical high-voltage electrode 22.5 cm in diameter was connected to a small copper loop through a ball chain. Corona discharge from these connections limited the attainable voltage to ~ 15 kV. Compressed air from a 2.5-cm-diameter tube behind the loop formed bubbles when the loop was periodically dipped in

[1] Report by Nezamaikin V N in [9].
[2] Nauer H, cited in [2], pp 164–5.

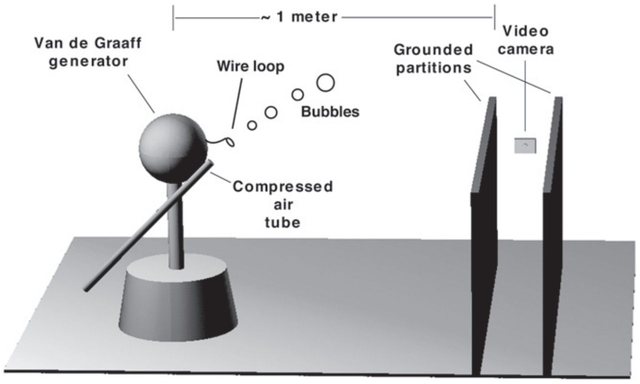

Figure 1. Layout of the charged-bubble experiment (not to scale) showing Van de Graaff generator and compressed-air bubble-producing mechanism on left, and trough with grounded bottom and side partitions on right. The trough was 40 cm wide by 60 cm high by 120 cm in length.

a soap solution. About 10% of the bubbles generated by the setup fell between the walls of a trough placed about 1 m away. The trough was formed of plastic-foam insulating material coated with aluminum foil. A video camera at one end of the trough photographed the bubbles in flight against a background of 2.5-cm spaced white gridlines on black cardboard at the other end of the trough. A second video camera (not shown) was placed at right angles to the long axis of the trough to locate the bubble's position along that axis. The inside surface of the trough wall nearest the Van de Graaff generator was electrically isolated and formed the input electrode of an electrometer. The electrometer input circuit was a 103 MΩ resistor in parallel with a 3.6-nF capacitor to ground. A high-impedance voltage follower (TL071CP) transmitted the electrometer voltage to a LabView$^{\mathrm{TM}}$ 12-bit A/D converter sampling at 1 kHz. This voltage was corrected for current offset errors and integrated in software to find the charge received at the electrometer input as a function of time.

Preliminary trials were performed using a captive bubble in a can-shaped electrode to verify that electrically isolated bubbles could retain their charge over the ~ 30 s time span of the experiment, and no observable leakage was detected. The raw electrometer output voltage during a trial in which five separate bubbles (or bubble clusters) collided with the electrometer wall is shown in figure 2. As a bubble approaches the wall, it induces charge of a polarity opposite to that of its own charge in the electrode by virtue of the increasing mutual capacitance between the bubble and the electrode. When the bubble contacts the wall, the capacitance goes to infinity and a peak occurs in the voltage, identifying the exact time of collision with a time resolution of 10 ms or better. The *R–C* parallel circuit's time constant $\tau = 0.33$ s shapes the discharge waveform as an exponential tail as shown. As long as bubble events are separated by 1 s or more, the voltage waveform $v(t)$ can be integrated during the time interval from t_1 to t_2 when the voltage v exceeds 10% of its peak value to obtain an estimate of the total bubble charge Q in accordance with the equation

$$Q \approx C[v(t_2) - v(t_1)] + \frac{1}{R} \int_{\tau=t_1}^{t_2} v(\tau)\,\mathrm{d}\tau. \tag{3}$$

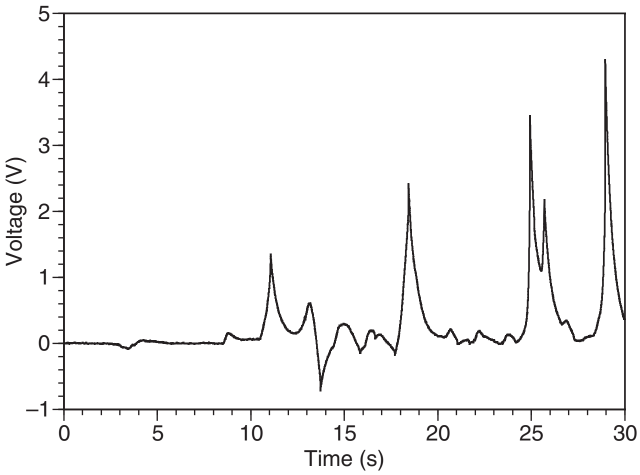

Figure 2. Raw voltage obtained from electrometer circuit during charged-bubble test. Bubble impacts are clearly visible at $t = 11.06$, 18.44, 24.93, 25.70 and 28.96 s, all correlating with videotaped impacts of single or multiple bubbles.

Although this procedure does not capture all the bubble's charge, it produces an estimate for Q that is accurate to within $\pm 10\%$ to $\pm 20\%$, depending on the waveform shape.

During the experiment, a significant difference was noted between the motion of charged and uncharged bubbles (made with the Van de Graaff generator off). Uncharged bubbles drifted aimlessly around the laboratory for as long as 30 s before either rupturing because of film evaporation or because of a collision with an object (typically the floor). All bubbles had a negative buoyancy, drifting straight down in the absence of other forces. However, charged bubbles moved more rapidly than uncharged ones. The charged bubbles were vigorously repelled from the region of the Van de Graaff generator and attracted to any grounded object such as a metal cabinet or stanchion. Although bubble lifetimes were not measured precisely, the average lifetime of the charged bubbles was probably less than half that of the uncharged bubbles.

To show that we can accurately model all forces involved in bubble motion in this experiment, we obtained three-dimensional position data on a bubble with a charge of $Q = -4.1$ nC as measured by its electrometer signal and a radius $r_0 = 1.6$ cm. Position data was obtained for the last 0.5 s or so of its lifetime up to the point when it collided with the electrode and ruptured. Independent measurements of the accuracy of our video system and data analysis procedure indicate that the probable rms measurement error is about 6 mm in the x-direction and 1.3 cm in the z-direction.

To see why bubbles are attracted to grounded surfaces, consider the idealized situation shown in figure 3, which shows a trough that is infinitely long in the y-direction (into the page) and infinitely tall in the z-direction (h goes to infinity). If a bubble of radius r_0 has a charge Q, that charge will induce a surface charge of the opposite polarity on the grounded surfaces surrounding the bubble. These charges in turn attract the bubble with a force that is balanced horizontally when $x = 0$ (in the center of the trough), but is always directed downward vertically, since there is no upper boundary to the trough. As the bubble approaches a side wall, the induced surface charge becomes unbalanced, becoming

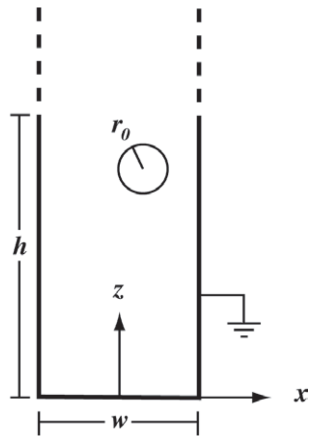

Figure 3. Charged bubble of radius r_0 in grounded conductive trough (trough is infinitely long in *y*-direction).

greater at the nearest wall and less at the opposite wall. This results in a net horizontal force which pushes the bubble toward the nearest wall until it collides with the wall and ruptures.

A concise summary of the forces on the bubble supports these ideas. The gravitational force on the bubble (in newtons) is

$$\vec{F}_G = -\hat{z}(4\pi r_0^2 \rho_F t g), \tag{4}$$

where $g = 9.8\,\mathrm{m\,s^{-2}}$ (the acceleration of gravity) and $\hat{z}$ is the unit vector directed vertically upward. The net gravitational force is due only to the mass of the bubble film of thickness t (m) and density ρ_F ($\mathrm{kg\,m^{-3}}$), since the mass of the entrapped air is canceled out by the buoyancy due to displacement of an approximately equal mass of air.

The electrostatic force on the bubble can be derived from a function $C(\vec{r})$ which is the capacitance between the bubble and the trough as a function of the bubble center's position vector $\vec{r}$. Since the bubble's electrostatic potential energy is

$$U_E(\vec{r}) = \frac{C(\vec{r})V^2(\vec{r})}{2}, \tag{5}$$

where $V(\vec{r})$ is the potential of the sphere (in volts), and for a constant charge $Q = C(\vec{r})V(\vec{r})$,

$$U_E(\vec{r}) = \frac{Q^2}{2} \cdot \frac{1}{C(\vec{r})}, \tag{6}$$

we use the fact that the negative gradient of the electrostatic potential energy is the electrostatic force

$$\vec{F}_E(\vec{r}) = -\frac{Q^2}{2}\nabla\left(\frac{1}{C(\vec{r})}\right). \tag{7}$$

In this way, the problem of finding the electrostatic force reduces to the problem of finding the bubble's capacitance to ground as a function of position. This analysis was performed with COMSOL finite-element software for the specific case of a trough with width $w = 40$ cm. The open trough was modeled by a closed prism 2 m high (z) by 1.9 m deep (y) by 40 cm wide (x). As long as the bubble was centered in the y-direction in the COMSOL model, we found that further increases of dimensions of the trough beyond those stated made less than 1% difference in the capacitance indicated by the model. This indicated that the model capacitance in the closed prism is a close approximation to the capacitance in the open laboratory trough. We explain below how we used data from this closed-prism model in the open-trough experimental situation. We then fit empirical analytical expressions to the COMSOL data for use in a differential-equation solution described below.

A bubble moving in air experiences a drag force

$$\vec{F}_D(\vec{v}) = -\vec{v}\frac{\pi r_0^2 \rho_{AIR}|\vec{v}|}{2} C_D(Rey(|\vec{v}|)), \tag{8}$$

where the velocity vector $\vec{v}$ ($\mathrm{m\,s^{-1}}$) is the time derivative of the position vector $\vec{r}$, the air density is ρ_{AIR}, and the dimensionless drag coefficient C_D is a tabulated function of the Reynolds number

$$Rey(|\vec{v}|) = \frac{2r_0\rho_{AIR}|\vec{v}|}{\mu}, \tag{9}$$

in which the dynamic viscosity of air is μ ($\mathrm{N\text{-}s\,m^{-2}}$) [18]. Analytic expressions approximating these functions in the region of interest were derived and used together with the electrostatic-force expressions to solve the overall second-order differential equation of motion for the bubble:

$$\vec{F}_G + \vec{F}_E(\vec{r}) + \vec{F}_D(\vec{v}) = m_B\frac{d^2\vec{r}}{dt^2}. \tag{10}$$

In equation (10), the mass m_B includes both the film mass and the mass of the gas inside the bubble:

$$m_B = 4\pi r_0^2\rho_F t + \tfrac{4}{3}\pi r_0^3 \rho_{AIR}. \tag{11}$$

Equation (10) was integrated using a Runge–Kutta algorithm and a time step of 11.1 ms for comparison with experimental data, which we will now describe.

3. Results

The theoretical and experimental results of the analysis of a charged bubble moving in the grounded trough are shown in figure 4. As the figure shows, we obtain good agreement between the actual measured path of the bubble as it is attracted to the electrode wall, and the theoretical prediction of the path based on the three forces in equation (10). Definite integration of equation (10) requires values for the bubble mass m_B and initial conditions for $\vec{v}$ and $\vec{r}$ at an initial time t_0. We obtained a value for the bubble mass by measuring its average velocity outside the trough from video data, since a bubble's terminal velocity (free-fall in a gravitational field only) depends sensitively on its mass. By the time bubbles reach the trough, they have lost any horizontal motion, so the initial velocity vector was known. The initial positions x_0 and y_0 were known from the video recordings. We chose the initial position for the theoretical curve to be $z_0 = 56.7$ cm, which yielded the best fit to the experimental trajectory. Since our COMSOL electrostatic model assumed an infinitely deep trough, it does not accurately model the situation above the edges of the laboratory trough, where the electrostatic force eventually goes to zero above the edges located at $z = 60$ cm. The choice of $z_0 = 56.7$ cm 'turns on' the electrostatic force at that height, and models the actual situation well.

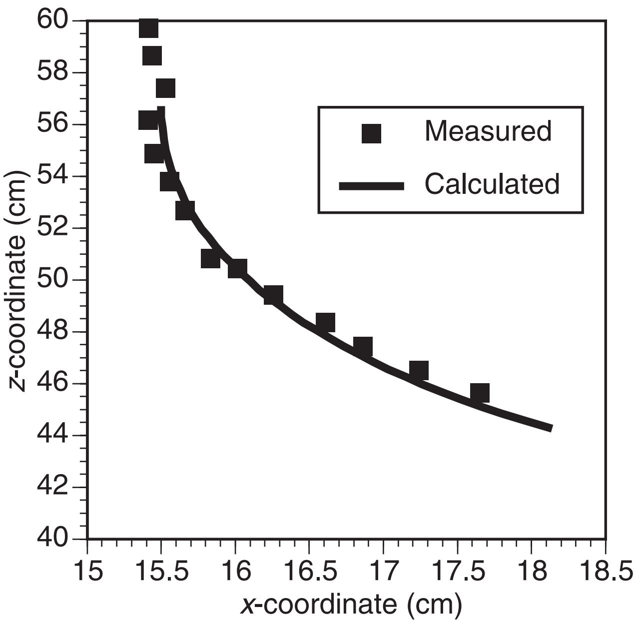

Figure 4. Theoretical and experimental trajectories of charged bubble near grounded wall at $x = 20$ cm with bubble radius $r_0 = 1.6$ cm and charge $Q = -4.1$ nC. Theoretical curve (solid line) begins at initial position $x_0 = 15.5$ cm and $z_0 = 56.7$ cm. The x-axis is drawn to a larger scale than the z-axis to make comparison easier.

4. Conclusions

Assuming a ball-lightning object is self-contained, and is not simply the visible manifestation of a larger independent field or current structure, its trajectory will be determined by the sum of the forces acting on it. This assumption is made in the two internal-energy theories summarized earlier in this paper. We have shown that soap bubbles charged to relatively small levels tend to be rapidly attracted to grounded surfaces and destroy themselves upon impact. We have also shown that the trajectory of a bubble can be accurately modeled by considering only drag, gravity, and the electrostatic force due to the bubble's capacitance to ground.

While care should be taken in extrapolating these results to larger objects farther away from grounded surfaces, the invariance of electrostatic solutions to Maxwell's equation under dilation (multiplication of all space dimensions by a constant) means that the same voltages and forces will apply to a scaled-up version of our experiment, as long as the charge on the object is scaled by the same factor. Scaling a 1.6-cm-radius bubble to 16-cm-radius (a typical size for ball lightning) means that a charge of the order of 40–120 nC will result in behavior similar to what we observed, although drag does not scale linearly with dilation as the electrostatic solution does. Therefore, a net electrostatic charge on a ball-lightning object of less than 0.2 μC or so is consistent with many observations, although charges near the upper end of this range would tend to shorten the phenomenon's lifetime through attraction to grounded objects. But a charge of 3 μC, as proposed by Turner, may approach the Rayleigh limit [19] at which an object held together by surface tension flies apart because of electrostatic repulsion. Calculating the Rayleigh limit requires a knowledge of the object's surface tension, which is not known for ball lightning. However, the charge limits we found were far below the Rayleigh limit for soap bubbles. The essential conclusion of this paper is that for ball lightning in the neighborhood of the earth's surface or other grounded objects, the upper limit for net electrostatic charge is established not by the Rayleigh limit at which the object self-destructs due to mutual repulsion, but by attraction to grounded objects through the mechanism of induced charge.

Even if a spherical object has no net charge, it can develop a dipole moment in a uniform electric field and experience a net attractive force in a non-uniform field. Since ball lightning is generally associated with thunderstorms and the electric field on the ground beneath such storms can greatly exceed the fair-weather field of about 100 V m^{-1}, it is possible that electrostatic forces are involved in ball-lightning motion near objects that would tend to concentrate electric field lines. This concentration leads to non-uniformities in the field and would produce a net force on a neutral conducting object such as ball lightning, assuming ball lightning has a non-zero conductivity. While we did not investigate this possibility, such problems may serve as a focus for further investigations of the problem of ball lightning motion in electrostatic fields.

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